INTRODUCTION TO MAPLE

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Old versions of Maple required all commands to be ended with a semicolon (`;`). Maple versions 10 and above are more flexible and will still understand what you mean if you leave the semicolon off. If you are using this hand-out with a version of Maple older than version 10, be sure all your Maple commands end with a semi-colon.

Maple has two modes, "document" mode and "worksheet" mode. This hand-out is intended for "worksheet" mode, but should also work in document mode.

Maple as a Calculator

Maple can do any of the computations that you could use a hand calculator to do. The basic arithmetic operations are `+` (addition), `-` (subtraction), `*` (multiplication), and `/` (division). For example, if you want to add 6 and 8, you would type

```
> 6+8;
```

and Maple will respond

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You would enter an expression into Maple, just like you would write it on paper. Maple does division and multiplication before addition and subtraction, so for instance if you type

```
> 2+3*5;
```

you get 17. If you want Maple to add 2 to 3 before multiplying by 5, you should include parentheses, just like you would on paper.

```
> (2+3)*6;
```
(You type)

30 (Maple's response)

Warning! Be careful not to leave out the mutiplication sign. Maple will not always give you an error, but also will not always give you the answer you expect. In Maple (versions 10 and higher), if you type `(2+4)(3+1)`, you get

```
> (2+4)(3+1);
```
(You type)

6 (Maple's response)

which is probably not what you want. On the other hand:

```
> (2+4)*(3+1);
```
(You type)

24 (Maple's response)

When you type `(2+4)(3+1)`, Maple is plugging in $3+1=4$ into the constant function $2+4=6$. Since anything plugged into the constant function $f(x)=6$ is always 6, it gives you 6 as an answer.

You can use the percent (`%`) key to refer to Maple's most recent response. So, for example, if you would like to add 10 to the previous line, you could type

```
> %+10;
```
(You type)

34 (Maple's response)

An important difference between Maple and a calculator is that Maple leaves all of its answers in

exact form, unless you tell it otherwise. For example if you type

```
> 2+15/7;
```
(You type)

$$\frac{29}{7}$$
(Maple's response)

Maple's response is a fraction. If you want Maple to give you a decimal approximation, you need to use the `evalf` command. For example, if you then type

```
> evalf(%);
```
(You type)

4.142857143 (Maple's response)

then Maple gives you the decimal approximation for 29/7.

Another important key you will need is the `^` key. This let's you raise something to a power. For example

```
> 2^3;
```
(You type)

8 (Maple's response)

Of course, Maple also has all the elementary functions built into it. For example, if you want to take a square root,

```
> sqrt(2);
```
(You type)

$$\sqrt{2}$$
(Maple's response)

Notice again that Maple's response is in exact form. If you want a decimal approximation, you need to use `evalf`.

```
> evalf(sqrt(2));
```
(You type)

1.414213562 (Maple's response)

If you want to compute the sine of an angle,

```
> sin(Pi/3);
```
(You type)

$$\frac{1}{2}\sqrt{3}$$
(Maple's response)

To compute e to some power, say e^2, you type

```
> exp(2);
```
(You type)

$$e^2$$
(Maple's response)

```
> evalf(%);
```
(You type)

7.389056099 (Maple's response)

Here are some more examples

```
> arctan(1);
```
(You type)

$$\frac{1}{4}\pi$$
(Maple's response)

```
> evalf(ln(2));
```
(You type)

.6931471806 (Maple's response)

Maple as More Than a Calculator

What makes Maple much more useful than a calculator is its ability to work with variables. For example,

```
> (1+2*x)*(3*x+4*x^2)+x^6-6*x^2;
```
(You type)

$$(1+2x)(3x+4x^2)+x^6-6x^2$$ (Maple's response)

An important command you might find useful is `simplify`. For example, to have Maple multiply out the above expression, you would type

```
> simplify(%);
```
(You type)

$$3x+4x^2+8x^3+x^6$$ (Maple's response)

In old versions of Maple, you always had to include the multiplication star (`*`) in your input. In older versions of Maple, Maple would not understand `2x` as `2*x`. Beginning with Maple 10, you may leave out the multiplication star before variables. **HOWEVER, it is not safe to leave out the multiplication star before parentheses.**

The `simplify` command does not always multiply out. It tries to put answers in what it considers to be a simple form. For instance if you type

```
> simplify(x*(x+1));
```
(You type)

$$x(x+1)$$ (Maple's response)

Maple will leave it alone. If you want to force Maple to multiply out, type

```
> expand(x*(x+1));
```
(You type)

$$x^2+x$$ (Maple's response)

If instead of multiplying out, you want Maple to factor, then you can use the `factor` command. For example,

```
> factor(x^3+7*x^2-x-7);
```
(You type)

$$(x+1)(x-1)(x+7)$$ (Maple's response)

Maple can even factor an expression with unspecified constants. For example,

```
> factor(x^3-a*x^2-x+a);
```
(You type)

$$-(x-1)(x+1)(-x+a)$$ (Maple's response)

Notice that if I expand an expression with several variables, Maple treats them all equally:

```
> expand((x-a)*(x-b)*(x-2));
```
(You type)

$$x^3-2x^2-x^2b+2xb-ax^2+2ax+abx-2ab$$ (Maple's response)

If you want to treat x as the variable, you can say

```
> collect(%,x);
```
(You type)

$$x^3+(-a-2-b)x^2+(2a+ab+2b)x-2ab$$ (Maple's response)

to "collect" the answer by powers of x.

Maple also allows you to assign values to symbols. For example, if you want to let a be the constant $\sqrt{5}$ you could type:

```
> a:=sqrt(5);
```
(You type)

$$a := \sqrt{5}$$ (Maple's response)

```
> x-a;
```
(You type)

$$x-\sqrt{5}$$ (Maple's response)

Notice that for assignment, you use `:=` and not just `=`.

To "unassign" the value to the symbol a, you would type:

```
> a:='a';
```
(You type)

$$a := a$$
(Maple's response)

You can undo all prior definitions by giving the `restart` command.

```
> restart;
```

Maple can solve equations for you. For example,

```
> solve(x^2-7*x+9=0);
```
(You type)

$$\frac{7}{2} + \frac{1}{2}\sqrt{13}, \frac{7}{2} - \frac{1}{2}\sqrt{13}$$
(Maple's response)

```
> evalf(%);;
```
(You type)

$$5.302775638,\ 1.697224362$$
(Maple's response)

Maple will even solve systems of equations for you. For example

```
> solve({x+y=1,x-y=7});
```
(You type)

$$\{y = -3, x = 4\}$$
(Maple's response)

You can even solve equations with unspecified constants:

```
> solve({x+y=a,x-2*y=b},{x,y});
```
(You type)

$$\{y = \frac{a}{3} - \frac{b}{3}, x = \frac{2a}{3} + \frac{b}{3}\}$$
(Maple's response)

The optional second argument to the `solve` function allows you to specify which variables to solve for. Here we are telling Maple that x and y are the variables we want to solve for.

We saw above that in Maple `Pi` is used to refer to the constant π. In Maple, the names of commonly used constants start with capital letters. For example `I` is used for the imaginary number i. For example,

```
> (1+I)*(1-I);
```
(You type)

$$2$$
(Maple's response)

```
> sqrt(-1);
```
(You type)

$$I$$
(Maple's response)

Functions

You can also define functions in Maple. If you want to work with the function $f(x) = x + \sin x$, you could type

```
> f := x-> x + sin(x);
```
(You type)

$$f := x \to x + \sin(x)$$
(Maple's response)

If you want to find out what $f(2)$ is, then you type

```
> f(2);
```
(You type)

$$2 + \sin(2)$$
(Maple's response)

Of course, if you want a decimal answer, you need to use `evalf`.

```
> evalf(%);
```
(You type)

$$2.909297427$$
(Maple's response)

An alternate approach to functions

If we enter

```
> f:=x^2+x;
```
(You type)

$$f := x^2 + x$$
(Maple's response)

then we are not really defining f as a function. We are defining f as an abbreviation for $x^2 + x$. Thus, entering `f(2)` does not do what we would expect:

```
> f(2);
```
(You type)

$$x(2)^2 + x(2)$$
(Maple's response)

To actually substitute 2 in for x, we could type

```
> subs(x=2,f);
```
(You type)

$$6$$
(Maple's response)

Since f is an abbreviation for x^2+x, entering `factor(f)` is the same as entering `factor(x^2+x)`.

```
> factor(f);
```
(You type)

$$x(x+1)$$
(Maple's response)

The function e^x.

The function e^x is denoted `exp` in Maple. Thus to find e^2 you would type:

```
> exp(2);
```
(You type)

$$\mathbf{e}^2$$
(Maple's response)

```
> evalf(%);
```
(You type)

$$7.389056099$$
(Maple's response)

To work with the number e, just use `exp(1)`. For example to find $\sqrt{e}$ you would do:

```
> sqrt(exp(1));
```
(You type)

$$\mathbf{e}^{\left(\frac{1}{2}\right)}$$
(Maple's response)

```
> evalf(%);
```
(You type)

$$1.648721271$$
(Maple's response)

Calculus Functions in Maple

To simplify your life even further, Maple can differentiate and integrate. For example, to have Maple compute the derivative of $f(x) = x^2 + \sin x$, you would type

```
> f:=x->x^2+sin(x);
```
(You type)

$$f := x \to x^2 + \sin(x)$$
(Maple's response)

```
> diff(f(x),x);
```
(You type)

$$2x + \cos(x)$$
(Maple's response)

or you could just type

```
> diff(x^2+sin(x),x);;
```
(You type)

$$2x + \cos(x)$$
(Maple's response)

to take the derivative without giving the function a name first. The second argument to the `diff` function tells Maple what letter is the variable. This lets you take the derivative of functions that also include constants. For example,

```
> f:=t->t^n + exp(a*t) + sin(omega*t);;
```
(You type)

$$f := t \mapsto t^n + e^{(at)} + \sin(\omega t)$$ (Maple's response)

```
> diff(f(t),t);
```
(You type)

$$\frac{t^n n}{t} + ae^{(at)} + \cos(\omega t)\omega$$ (Maple's response)

Notice that you can get Greek letters just by spelling them out, and that sometimes it might be necessary to add a `simplify` command after taking derivatives to get Maple to simplify its answer.

Now suppose you want to compute a definite integral like

$$\int_1^3 x^2\,dx.$$

To enter this into Maple, you would just type

```
> int(x^2,x=1..3);
```
(You type)

$$\frac{26}{3}$$ (Maple's response)

The first argument to `int` is the function you want to integrate, and the second argument tells Maple both what the variable is, and what the limits of integration are. Notice that Maple gives you an exact answer; if you want a decimal approximation, you need to use `evalf` as always. For instance

```
> int(cos(x),x=0..1);
```
(You type)

$$\sin(1)$$ (Maple's response)

```
> evalf(%);
```
(You type)

.8414709848 (Maple's response)

Of course, not all functions can be integrated in terms of elementary functions, as in the following example.

```
> int(exp(x^3),x=0..1);
```
(You type)

$$\int_0^1 e^{(x^3)}\,dx$$ (Maple's response)

When this happens, Maple just gives you the integral back. However, you can still get a decimal approximation by typing `evalf`.

```
> evalf(%);
```
(You type)

1.341904418 (Maple's response)

Of course, Maple has no problem doing integrals with constants. If you want to compute $\int_1^{b^2} e^{ax}\,dx$, you could just type

```
> simplify(int(exp(a*x),x=1..b^2));
```
(You type)

$$\frac{e^{(ab^2)} - e^a}{a}$$ (Maple's response)

If you want Maple to give you an anti-derivative (*i.e.* compute an indefinite integral) then you just make the second argument of `int` the variable, without the range. For example

```
> f:=x->1/sqrt(1-x^2);
```
(You type)

$$f := x \to \frac{1}{\sqrt{1-x^2}}$$ (Maple's response)

```
> int(f(x),x);;
```
(You type)

$$\arcsin(x)$$
(Maple's response)

To evaluate multiple integrals, just put one integral inside another. For example,

```
> Int(Int(Int(x*(y^2+z^3),x=-2*y..4*z),y=-z..4),z=-2..2);
```
(You type)

$$\int_{-2}^{2}\int_{-z}^{4}\int_{-2y}^{4z} x(y^2+z^3)dxdydz$$
(Maple's response)

Many Maple commands, like `int`, come in two versions. The command `Int` starting with a capital letter delays or avoids evaluation of the command. Why these commands exist is beyond the scope of this elementary introduction. I did this here only to illustrate what the command would compute. To actually evaluate the integral, type the command without the capital `I`.

```
> int(int(int(x*(y^2+z^3),x=-2*y..4*z),y=-z..4),z=-2..2);
```
(You type)

$$\frac{-144896}{315}$$
(Maple's response)

Maple as a Graphing Tool

We will begin by plotting the graph of a function. For example if you want to see what the graph of the function $y = e^{x/3}\sin(3x)$ looks like between $x = 0$ and $x = 10$, you would type

```
> plot(exp(-x/3)*sin(3*x),x=0..10);
```

To plot several functions together, you can do the following:

```
> plot({x,x^2,sin(x)},x=0..5);
```

To make a point plot, you could type:

```
> plot([[1,2],[2,5],[4,6],[5,7]],style=point);
```

To plot a parametric curve:

```
> plot([sin(3*t),cos(5*t),t=0..2*Pi]);
```

There are a number of options you can adjust to change the look of your graph. Type

```
> ?plot[options];
```

to read the help screen describing the plot options. Here is an example of setting some of the style options:

```
> plot(x^2+x+1,x=0..3,color=red,view=[-1..4,-2..10],labels=['x','C'],
title="A Simple Plot",legend="cost");
```

If you use the plots library, invoked by the command

```
> with(plots);
```

there are many many more plotting and animation options. Type

```
> ?plots;
```

for more information. For example, the `animate` and `plot3d` commands are useful.

```
> animate(sin(x+t),x=-2*Pi..2*Pi,t=0..2*Pi);
> plot3d(sqrt(x^2+y^2),x=-3..3,y=-3..3,view=[-3..3,-3..3,-1..5],
scaling=CONSTRAINED, axes=FRAMED);
```

The "Plot Builder" assistant under the "tools" menu is also helpful for learning the plot commands.

Basic Differential Equations

Maple can solve simple ordinary differential equations. For example, you can enter a differential equation and solve it with:

```
> de:=diff(y(x),x)=2*y(x);
```
(You type)

$$\frac{d}{dx}y(x) = 2y(x)$$ (Maple's response)

```
> dsolve(de);
```
(You type)

$$y(x) = \_C1\,\mathbf{e}^{(2x)}$$ (Maple's response)

You can even put in initial conditions:

```
> dsolve({de,y(0)=3});
```
(You type)

$$y(x) = 3\mathbf{e}^{(2x)}$$ (Maple's response)

Of course many differential equations do not have solutions with nice formulas. For example, consider

```
> de:=diff(y(x),x$3)=sin(x)*y(x);
```
(You type)

$$de := \frac{d^3}{dx^3}y(x) = \sin(x)y(x)$$ (Maple's response)

```
> dsolve(de);
```
(You type)

$$y(x) = \mathrm{DESol}\left(\left(\left(\frac{d^3}{dx^3}\_Y(x)\right) - \sin(x)\_Y(x)\right), \{\_Y(x)\}\right)$$ (Maple's response)

This last response is Maple giving up on finding a formula for the solution. It is still possible to graph a solution though.

```
> with(DEtools);
> DEplot(de,y(x),x=0..5,[[y(1)=1,D(y)(1)=2,D(D(y))(1)=-1]],y=0..5);
```

This plots the solution to the initial value problem:

$$y''' = \sin(x)y \qquad y(1) = 1 \qquad y'(1) = 2 \qquad y''(1) = -1.$$

The command

```
> DEplot(diff(y(x),x)=y(x)*(1-y(x)),y(x),x=0..10,[[y(0)=0.2]],y=-0.5..1.5);
```

will plot a slope field for the logistic equation and the solution tranjectory satisfying the initial condition $y(0) = 0.2$.

Matrices and Linear Algebra

Maple has multiple add-in libraries to work with matrices and do computations related to linear algebra. The library that is the most student-oriented and probably goes along best with a typical sophomore linear algebra class is the `LinearAlgebra` library.

```
> with(LinearAlgebra);
```

Consider a system of equations:

$$x + y + z = 3 \qquad x - y + 2z = 5 \qquad 2x + y - z = 12.$$

We can have Maple convert this to an augmented matrix by using the command:

```
> M:=GenerateMatrix([x+y+z=3,x-y+2*z=5,2*x+y-z=12],[x,y,z],
augmented=true);
```
(You type)

$$M := \begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 1 & 2 & 5 \\ 2 & 1 & -1 & 12 \end{bmatrix}$$ (Maple's response)

You can immediately find the reduced row echelon form of the matrix by giving the command:

```
> ReducedRowEchelonForm(M);
```
(You type)

$$\begin{bmatrix} 1 & 0 & 0 & \frac{43}{7} \\ 0 & 1 & 0 & \frac{-12}{7} \\ 0 & 0 & 1 & \frac{-10}{7} \end{bmatrix}$$ (Maple's response)

You can do elementary row operations one at a time by using the `RowOperation` command. For example, `RowOperation(M,3,10)` multiplies Row 3 by 10:

```
> RowOperation(M,3,10);
```
(You type)

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & -1 & 2 & 5 \\ 20 & 10 & -10 & 120 \end{bmatrix}$$ (Maple's response)

The command `RowOperation(M,[1,3])` interchanges rows 1 and 3:

```
> RowOperation(M,[1,3]);
```
(You type)

$$\begin{bmatrix} 2 & 1 & -1 & 12 \\ 1 & -1 & 2 & 5 \\ 1 & 1 & 1 & 3 \end{bmatrix}$$ (Maple's response)

The command `RowOperation(M,[3,1],-2)` subtracts two times Row 1 from Row 3 and puts that in Row 3:

```
> RowOperation(M,[3,1],-2);
```
(You type)

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & -1 & 2 & 5 \\ 0 & -1 & -3 & 6 \end{bmatrix}$$ (Maple's response)

Maple can take the determinant of a matrix:

```
> M:=Matrix([[1,1],[1,-1]]);
```
(You type)

$$M := \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$
(Maple's response)

```
> Determinant(M);
```
(You type)

$$-2$$
(Maple's response)

Maple can find eigenvalues and eigenvectors:

```
> Eigenvalues(M);
```
(You type)

$$\begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \end{bmatrix}$$
(Maple's response)

```
> Eigenvectors(M);
```
(You type)

$$\begin{bmatrix} \sqrt{2} \\ -\sqrt{2} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}-1} & \frac{1}{-\sqrt{2}-1} \\ 1 & 1 \end{bmatrix}$$
(Maple's response)

So the eigenvalues are $\sqrt{2}$ and $-\sqrt{2}$. An eigenvector with eigenvalue $\sqrt{2}$ is the first column of the second matrix, namely

$$\begin{bmatrix} \frac{1}{\sqrt{2}-1} \\ 1 \end{bmatrix}.$$